SET - 1 **GR 14** CODE: GR14A1003

I B. Tech II Semester Regular Examinations, June, 2015
Transform Calculus and Fourier Series
(Common to CE, EEE, ME, ECE, CSE, BME and IT)

Time: 3 hours **Max Marks: 70**

PART – A

Answer ALL questions. All questions carry equal marks

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10 * 2 Marks = 20 Marks

1). a Define the Gamma function. Evaluate $\Gamma\left(\frac{3}{2}\right)$ **[2]**

b Find the Z transform of the sequence {2, -2, 2, -2, . . . } **[2]**

c A function f(t) is defined as $f(t)=\begin{cases}1 & 1<t<2\\ 0 & elsewhere\end{cases}$. Find its Laplace Transform **[2]**

d State the Parseval's identity in the context of Fourier Series of a periodic function f(x) defined over the range $-\pi \le x \le \pi$. **[2]**

e Define the Fourier Transform of $f(x)$ *in the range* $(-\infty,\infty)$. What is the corresponding inversion formula? **[2]**

f Solve the partial differential equation $x^2 p + y^2 q = z^2$, where p *and* q have their usual meanings. **[2]**

g Define the Beta function $\mathrm{B}(m,n)$. Express the integral $\int_0^1 \sqrt{x}(1-x)^{3/2}\,dx$ as a Beta integral. (You need not evaluate) **[2]**

h Find the transfer function $U(z)$ of the recurrence relation $u_{n+2}+6u_{n+1}+8u_n=0$ subject to the conditions $u_0=0$ *and* $u_1=-1$ **[2]**

i Evaluate $L^{-1}\left(\frac{s\,e^{-s}}{s^2+4}\right)$ stating the property that is required to evaluate it. **[2]**

j Write the formulae to compute the Fourier coefficients of a periodic function $f(x)$ defined over the range $(0,2l)$. **[2]**

PART – B

Answer any FIVE questions. All questions carry equal marks.

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5 * 10 Marks = 50 Marks

2. Use appropriate definitions of Gamma and Beta functions to establish the relation $\mathrm{B}(m,n)=\frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$. Evaluate the improper integral $\int_0^1 x^{-3/4}(1-x)^{-1/4}\,dx$. Do you need any special property for this evaluation? If so, state that property. **[10]**

3. (a) (i) Using the first shifting property evaluate $L\left(2\sinh 3t.\cos 4t\right)$ [2] **[10]**

(ii) Evaluate the improper integral $\int_0^\infty \frac{\sin 2t}{t}\,dt$ using the Laplace Transform [3]

(b) Evaluate $L^{-1}\left(\frac{s-5}{(s+2)\left(s^2+6s+13\right)}\right)$ using the method of partial fractions [5]

4. (a) Develop the half range sine series of the function $f(x)=2x-x^2$ *valid for* $0\le x\le 2$. **[10]**

Determine the value to which the infinite series $\frac{1}{1^3}-\frac{1}{3^3}+\frac{1}{5^3}-\frac{1}{7^3}+\ldots$ converges.

(b) Evaluate the initial sequence terms u_0 *and* u_1 if $Z(u_n)=U(z)=\frac{2z^3+3z^2+5z}{(z^2-4)(z^2+9)}$

5. Using the definition of the Fourier transform of f(x) as **[10]**

$F(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x).e^{-i\omega x}\,dx,$

find the Fourier transform of $f(x)=\begin{cases}1 & -a<x<a\\ 0 & elsewhere\end{cases}$

6. (a) Develop a homogeneous second order partial differential equation by eliminating the arbitrary functions in $z=f(y+3x)+g(y-6x)$ **[10]**

(b) Use the method of separation of variables to solve the first order partial differential equation $v_x+4v_y=3v$ subject to the condition $v(x,0)=10e^{-8x}$

7. Solve the initial value problem $y''(t)+4y(t)=e^{-t}\cos 3t$ subject to the initial conditions $y(0)=y'(0)=0$ using the Laplace transform. **[10]**

8. (a) A rod of length *10* cm has its ends insulated. An initial temperature $u(x)=u_0$ exists in the rod. Write the partial differential equation governing the heat flow in the rod. Clearly state the initial and boundary conditions that apply in this context. (You need not solve) [3] **[10]**

(b) Identify the partial differential equation $u_{xx}+u_{yy}=0$. Solve this equation subject to the following boundary conditions

$u(0,y)=u(8,y)=u(x,\infty)=0^0$ *and* $u(x,0)=50\sin\frac{5\pi x}{8}$ [7]

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